Interconnectedness and systemic risk in the US CDS market

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Abstract

This study assesses systemic risk in the US credit default swap (CDS) market. After the bankruptcy of Lehman Brothers, the market introduced risk mitigation tools, such as central clearing and portfolio compression in addition to existing netting and collateralization. Because CDSs typically have been traded as over-the-counter derivatives, just after the bankruptcy, few contracts were through central clearing, whereas in the first half of 2015, this share increased to 26%. First, we estimate the bilateral exposures matrix using aggregate fair value data on Call Reports by the Federal Deposit Insurance Corporation (FDIC) and theoretically analyze interconnectedness in the US CDS network using various network measures. The robustness of the estimated bilateral matrix is fully assured by sensitivity analysis using a core–periphery model and modified Jaccard index. Second, we theoretically analyze the contagious defaults introducing the Eisenberg and Noe framework. The network analysis shows that three to six dealers were central in the network in the past. The default analysis shows the theoretical occurrence of many stand-alone defaults and one contagious default via the CDS network during the global financial crisis. A stress test based on a hypothetical severe stress scenario predicts almost no future contagious defaults. To conclude, the risk contagion via the CDS network is unlikely.

Keywords: credit default swap; systemic risk; contagious default; interconnectedness; centrality measure
JEL classification: G10; D85; L14; G28; F37

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1. Introduction

There has been growing interest in the systemic importance of financial institutions stemming from credit default swap (CDS) contracts among US market regulators. Hence, the lack of both theoretical and empirical studies on the role of CDSs and their interconnectedness has become a major issue. The purpose of this study is to analyze the systemic importance of CDS market dealers in the US financial market.[1] Network analysis plays an important role in the analysis of systemic importance.

During the global financial crisis, CDSs became known as derivatives that triggered the systemic contagion risks in the derivative market. A typical example of contagions related to CDS contracts is the management crisis of American International Group (AIG), which was a major seller of CDSs. However, the AIG crisis was actually caused by a London-based derivative subsidiary – AIG Financial Products (AIG–FP), most of whose counterparties were Western financial institutions, such as Société Générale, Goldman Sachs, Deutsche Bank, Merrill Lynch, Calyon, UBS, and Deutsche Zentral-Genossenschaftsbank (Coral Purchasing) (ECB, 2009).

As at the end of September 2008, the aggregate gross notional amount of credit derivatives sold by AIG was 493 billion US dollars or 372 billion US dollars on a net basis. This amount potentially could affect the whole financial network. The net notional amount was almost double the aggregate net notional amount sold by all Depository Trust and Clearing Corporation (DTCC) dealers combined at the end of October 2008. However, as at the end of 2006, AIG was not ranked among the largest CDS market dealers in a Fitch survey, because AIG mainly sold bespoke CDSs, which were not covered by the DTCC data. Finally, as federal assistance to AIG, almost 50 billion US dollars was paid to the CDS counterparties at the end of 2008 (Harrington, 2009).

Since before the global financial crisis, the CDS market has had risk mitigation tools, such as netting and collateralization. Thereafter, significant contributions have been made with the aim of standardizing contracts, especially for the use of central counterparties (CCPs), including portfolio compression[2] and the International Swaps and Derivatives Association

[1] Major CDS dealers in the US CDS market are listed as G14 in Table A.6.

[2] Such companies as TriOptima provide compression services of CDS contracts (Gregory, 2014). This has been instrumental in reducing exposures.

(ISDA) "Big Bang" CDS Protocol in 2009. However, CDSs have typically been traded as over-the-counter (OTC) derivative instruments. At the end of 2010, while there was central clearing of a large proportion of all standing OTC interest rate products, mainly swaps, less than 10% of CDSs (on the basis of notional amounts outstanding) were cleared through central counterparties (Gregory, 2014; BIS, 2016). In the aftermath, at the end of June 2015, the share increased to 31% (BIS, 2016).

First, we analyze interconnectedness in the US CDS market using network centrality measures ("network analysis") (Jackson, 2010). We use the aggregate fair value data before considering collateralization on Call Reports by the FDIC. Our dataset covers most contracts by counterparties in the US CDS market. By contrast, because the other dataset from the DTCC contains the names of reference entities but the identity of the counterparties is anonymized, it is not available for our network analysis. From our analysis result, the US CDS market was composed of 10–20 major players, including dealer banks and non-dealer banks, during 2006–2015 and most contracts were executed via five dealers—JPMorgan Chase, Bank of America, Citibank, Goldman Sachs, and HSBC Bank USA.

Second, we conduct a model analysis of contagious defaults, applying the Eisenberg–Noe framework (Eisenberg and Noe, 2001) to the US CDS network and setting up a default mechanism ("default analysis"). In this framework, defaults are classified into stand-alone defaults and contagious defaults, which are defaults that trigger a domino effect. In the original Eisenberg–Noe framework, multi-step defaulting events are not expressed. Because contagious defaults can be caused in a specific setting, it is important to determine whether this framework is suitable for the theoretical analysis using real-world data. Strictly speaking, this framework describes only simultaneous defaults for one period, not a dynamic setting for a multi-period context. We use the framework in the multi-period setting using continuously estimated market values of assets and theoretically analyze the US CDS network. In addition, it is important to validate whether many defaulting and non-defaulting banks suffered losses owing to the payment defaults by the defaulting counterparties.

In addition, we conduct a stress test to assess the occurrence possibilities of contagious defaults in the future. An important preparation to conduct these analyses is the estimation of bilateral exposures in the US CDS market. To check the robustness of the estimated matrix, we conduct sensitivity analysis. Our methodology could assist in the development of monitoring and early-warning indicators related to systemic risk in the CDS network by supervisory authorities. Furthermore, it could be used in the implementation of banks' internal systemic stress tests of default contagion.

The rest of this paper is organized as follows. Section 2 reviews the extant literature on systemic risk in the CDS market and the interconnectedness in financial networks. Section 3 describes the network structure and the mechanism of defaults in the CDS market, and Section 4 describes the data used in this study. Section 5 discusses the estimations of the bilateral exposures matrix and market value of assets. Section 6 presents the results of the risk analysis. Section 7 assesses the robustness of the estimated bilateral exposures matrix, and Section 8 concludes.

2. Literature review

Brunnermeier et al. (2013) assess the potential systemic and contagion risks arising from a credit event for a major CDS reference entity or from the default of a key player in the CDS market. In particular, they argue that the multi-faceted nature of interconnectedness is difficult to capture in existing analytical frameworks. First, to understand risk transfer and risk-bearing capacity better, it would be necessary to know whether CDS exposures stem from proprietary trading, market making, or hedging. Second, counterparty credit risk is material in other OTC derivatives markets, in which the transaction volume cleared by CCPs is still relatively small.

Peltonen et al. (2014) analyze the network structure of the CDS market, using a sample of counterparties' bilateral notional exposures to CDS on 642 sovereign and financial reference entities as of the end of 2011. They use the dataset obtained from the DTCC to analyze the CDS exposures network from three different perspectives: (i) the "aggregated" CDS network,[3] (ii) sub-networks with a lower "aggregated" level, such as the sovereign CDS network, and (iii) networks for CDS reference entities. Their dataset contains the names of reference entities, but the identity of the counterparties is anonymized. Regarding the determinants of the network structure, they find a significant impact of the characteristics of the underlying bond exposure (size and collateralization) and of the risk characteristics of the CDS

[3]They define the aggregated CDS network using an unweighted adjacency matrix. Hence, some metrics, such as diameter, mean degree, and weighted degree, are difficult to calculate for a weighted network.

(volatility and commonality in returns) on the market size and activity of a given CDS reference entity.

Markose et al. (2012) provide an empirical reconstruction of the US CDS network for the fourth quarter in 2007 and 2008. The propagation of financial contagion in networks with dense clustering that reflects high concentration or localization of exposures between few players is identified as "too interconnected to fail." Systemic risk management from bank failures in uncorrelated random networks is different from those with clustering. Because systemic risk of highly connected financial institutions in the CDS markets is not priced into their holding of capital and collateral, they design a super-spreader tax based on eigenvector centrality of the banks that can mitigate potential socialized losses. Eigenvector centrality is one of the centrality measures and expresses the influence of a node in a network.

Network analysis is a highly effective approach to examine interconnectedness in CDS markets, which represent complex contract networks, with a set of "nodes" connected by "edges." In a CDS network, the nodes represent players and the edges represent the CDS contracts between the players.

An analysis of CDS networks would alert the supervisory authorities or individual financial institutions about "contagion risk" from the channels through which shocks propagate. Hence, the resilience of a network is tested in such analyses, and systemically significant nodes are identified. In addition, network analysis provides an empirical tool to test the effectiveness of macro-prudential policies.

An analytically tractable example in financial networks is the interbank network, which is characterized by bilateral exposures in the interbank market. In many countries, data pertaining to bilateral exposures are not published, and many researchers are unable to use these data. This difficulty is the same as that with CDS exposures data. Therefore, estimating the bilateral exposures matrix with the elements exposed from one bank to another is a significant challenge. Recently, some studies have adopted an information theory-based method that minimizes the amount of information required in the bilateral exposures matrix (e.g., Censor and Zenios, 1998; Sheldon and Maurer, 1998; Upper and Worms, 2002; Wells, 2004).

The extant literature on financial networks includes two approaches. The first describes the network structure using topological indicators. The literature often relates these indicators to model graphs based on network theory. This approach does not assume a mechanism by which shocks propagate within the network; therefore, it is referred to as "static network analysis"

(Alves et al., 2013). Eisenberg and Noe (2001), Boss et al. (2004), Afonso et al. (2011), Puhr et al. (2012), Tirado (2012), and Kanno (2015, 2016) are examples of studies based on this approach. Using the Austrian central credit register, Boss et al. (2004) and Puhr et al. (2012) report that the Austrian interbank market is hierarchized, and banks within subsectors tend to cluster together. The hierarchization of core banks and peripheral banks is confirmed for several national interbank systems, such as in Belgium (Degryse and Nguyen, 2007), Germany (Craig and von Peter, 2014), Italy (Iori et al., 2008), the Netherlands (In't Veld and Van Lelyveld, 2012), and the United Kingdom (Langfield et al., 2014).

The second approach assesses the strength of the contagion channels and the resilience of the network by observing the responses of financial network structures to shocks. The introduction of a shock assumes a specific transmission mechanism, such as defaults by market dealers. This approach is referred to as "dynamic network analysis" in Alves et al. (2013). Some extant studies that focus on the analysis of contagion effects include Elsinger et al. (2006), Cocco et al. (2009), Haldane and May (2011), and Duan and Zhang (2013).

Eisenberg and Noe (2001) develop a fundamental framework for assessing contagious default. According to their theorem, under mild regularity conditions, a unique "clearing payment vector" exists that clears members' obligations from the clearing system. However, because no closed-form solution exists for the distribution of the payment vector in this algorithm, a simulation approach based on hypothetical scenarios must be used. The model discussed in Section 3 provides details about this approach.

3. Network structure and default mechanism

In this section, we describe the network structure of the US OTC CDS market and the mechanism of contagious defaults in the market.

3.1. Network structure

CDSs typically are traded as OTC derivative instruments. Before the global financial crisis, OTC clearing mostly was conducted bilaterally between the dealers involved (left panel of Figure 1). By contrast, central clearing has been introduced in 2009 (right panel of Figure 1). BIS statistics (BIS, 2016) shows that the outstanding contracts held with CCPs at the end of June 2010 accounted for 9.8% of the notional amounts of outstanding

CDSs, whereas they accounted for only 3.8% of the outstanding gross market values (GMV); at the end of June 2015, each share had monotonically increased to 31% and 26%, respectively. The time-series transitions for the share with banks (dealer banks and other banks) and the share with CCPs after the first half of 2010 are shown in Figure 2. We recognize that the decrease in the share with banks results in the increase in share with CCPs.

The CDS market players comprise dealers and non-dealers. The non-dealers include pension funds, asset managers, hedge funds, other banks (except dealer banks), and non-financial companies. In terms of number of trades and notional amounts outstanding, shares for dealer–dealer in the US OTC CDS market are 83.7% and 81.6%, respectively (Table 1). Hence, we deal with dealer–dealer trades and other bank trades included in non-dealer trades in relation to the network structure of the US OTC CDS market, although our dataset from FDIC Call Reports does not distinguish between them.

After the bankruptcy of Lehman Brothers, two CCPs—ICE Clear Credit[4] and CME Group[5]—offer central clearing functions in the US OTC CDS market. ICE Clear Credit is a major clearing house whereas CME Group's volumes have been small (Gregory, 2014). In central clearing, because a CCP stands between market dealers, it has a matched book and takes on the counterparty risk.

Hence, we solve the payments for the bilateral market without central clearing because in the central clearing market, a counterparty to a dealer is a CCP that takes responsibility for closing out all the positions of a defaulting clearing member.

3.2. Default mechanism

We assume that the market value-based balance sheet is composed of the market value-based asset and equity values. This sheet is divided into an

[4]ICE Clear Credit is a dedicated CDS clearing house based in the US. It was launched in March 2009 as ICE Trust US and was regulated as a bank by the New York State Banking Department and the Federal Reserve Board of Governors. The clearing house has offered indirect clearing for buy-side firms since December 2009. On July 16, 2011, in accordance with the Dodd–Frank Wall Street Reform and Consumer Protection Act, the clearing house became a CFTC-regulated derivatives clearing organization and SEC-regulated securities clearing agency and changed its name to ICE Clear Credit.

[5]Dealer names in two CCPs are listed in Table A.6.

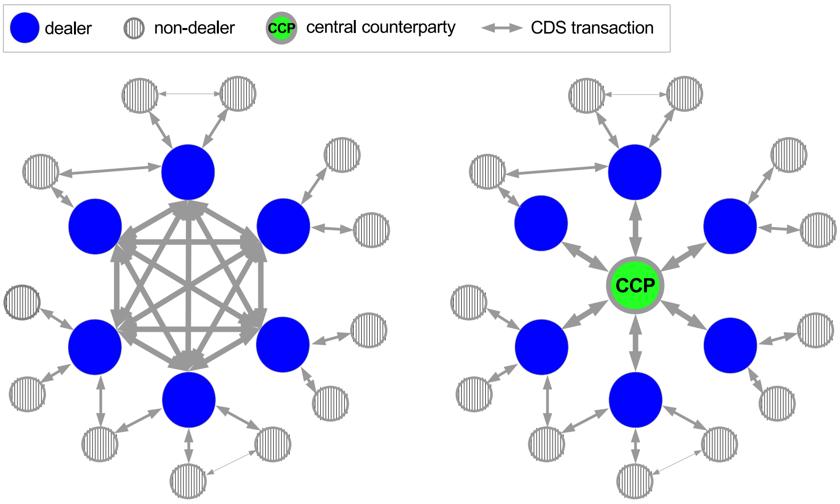

Figure 1: The network structure of the US OTC CDS market

Note: left panel: bilateral market; right panel: central clearing market.

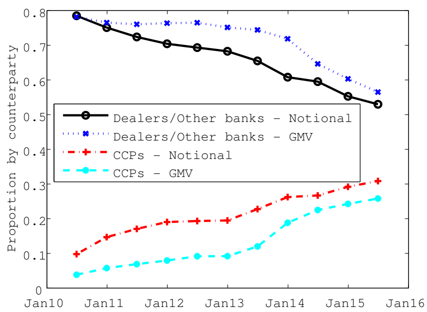

Figure 2: Shares in the global OTC CDS market

Note: All figures are adjusted for double counting. Gross market values (GMV) have been calculated as the sum of the total "gross positive market value" of contracts and the absolute value of the "gross negative market value" of contracts with non-reporting counterparties.

Source: BIS statistics.

Table 1: Dealer/non-dealer trading volumes in the US OTC CDS market (unit amount: in billions of US dollars)

	No. of trades	Notional amounts
Dealer – dealer trades	1,798,455	12,142
Non-dealer sells to dealer	155,723	1,266
Non-dealer buys from dealer	193,235	1,449
Non-dealer – non-dealer trades	2,470	24
Subtotal: Non-dealer trades	351,428	2,739
Total	2,149,883	14,881
Dealer – dealer share	83.7%	81.6%
Non-dealer related share	16.3%	18.4%

Note: The data are for the week of April 26–30, 2010.

Source: DTCC and Kamakura Corporation (Kamakura, 2010).

item in response to CDS payments and the other items. To this end, we apply the fundamental framework proposed by Eisenberg and Noe (2001) for risk analysis of the OTC CDS market. Following their framework, we simultaneously solve CDS payments of all the players in the market. The monthly solutions are obtained using the market value-based parameters of assets and volatilities estimated from a continuous-time model.

Consider a set of CDS market dealers $\mathcal{N} = \{1, \ldots, N\}$ at time $t \in [0, T]$. The CDS network structure is represented as (L, e), where $L = (l_{ij})_{1 \leq i,j \leq N}$ is an $(N \times N)$ CDS bilateral exposures matrix, and e is the exogenous net claims cash flow vector, which is calculated as the difference between the market value of assets and the book value of liabilities.

If the total value of a CDS player becomes negative for a pair (L, e), the dealer becomes insolvent. Let $d_i = \sum_{j=1}^{N} l_{ij}$ represent the total obligations of dealer i to all players j (any $j \in \mathcal{N}$) of the network. In addition, we consider a matrix $\Pi \in [0, 1]^{N \times N}$, which is derived by normalizing the entries with the total obligations:

$$\Pi_{ij} = \begin{cases} \frac{l_{ij}}{d_i} & \text{, if } d_i > 0 \\ 0 & \text{, otherwise.} \end{cases} \tag{1}$$

For any $d_i > 0$, an expected CDS payment claims row vector $d^{'}\Pi$ is as follows:

$$d^{'}\Pi = \begin{bmatrix} d_1 & d_2 & \cdots & d_N \end{bmatrix} \begin{bmatrix} \frac{l_{11}}{d_1} & \cdots & \frac{l_{1j}}{d_1} & \cdots & \frac{l_{1N}}{d_1} \\ \vdots & \ddots & \vdots & \iddots & \vdots \\ \frac{l_{i1}}{d_i} & \cdots & \frac{l_{ij}}{d_i} & \cdots & \frac{l_{iN}}{d_i} \\ \vdots & \ddots & \vdots & \iddots & \vdots \\ \frac{l_{N1}}{d_N} & \cdots & \frac{l_{Nj}}{d_N} & \cdots & \frac{l_{NN}}{d_N} \end{bmatrix} \tag{2}$$
$$= \begin{bmatrix} \sum_{i=1}^{N} l_{i1} & \sum_{i=1}^{N} l_{i2} & \cdots & \sum_{i=1}^{N} l_{iN} \end{bmatrix}.$$

A CDS network is described as a 3-tuple (Π, e, d), for which we define a clearing payment vector p^*. The clearing payment vector represents the limited liabilities of the dealers and the proportional sharing in the event of a default. A payment vector $p^* \in [0, d]$ is a clearing payment vector if and only if the following condition holds:

$$p_i^* = \begin{cases} d_i & \text{, case 1: if } \sum_{j=1}^{N} \Pi_{ij}^{'} p_j^* + e_i \geq d_i \\ \sum_{j=1}^{N} \Pi_{ij}^{'} p_j^* + e_i & \text{, case 2: if } 0 \leq \sum_{j=1}^{N} \Pi_{ij}^{'} p_j^* + e_i < d_i \\ 0 & \text{, case 3: if } \sum_{j=1}^{N} \Pi_{ij}^{'} p_j^* + e_i < 0 \end{cases} \tag{3}$$

The condition for case 1 is a solvent case for bank i. By contrast, the conditions for cases 2 and 3 are defaulting cases. The loss vector for the dealers is calculated as follows:

$$loss := \Pi^{'} d - \Pi_{new}^{'} p^*. \tag{4}$$

The matrix Π_{new} is defined in the following equation (6). We adopt the default algorithm developed by Eisenberg and Noe (2001) to find a clearing payment vector. They prove that a unique clearing payment vector always exists for (Π, e, d) under mild regularity conditions. These results apply to our multi-period framework.

The number of defaulting players is calculated by comparing the clearing payment vector with the current payment vector. A theoretical default algorithm is implemented to calculate the clearing payment vector, which is summarized as follows:

Step 1: Initialize $p_j = d_j$ and calculate the net claim value of bank j as

$v_j := \sum_{m=1}^{N} \Pi'_{jm} p_m + e_j - d_j$. If $v_j > 0$, its bank does not default, the clearing payment vector is $p_j = d_j$, and this algorithm ends. Otherwise, proceed to Step 2.

Step 2: Find banks with net value $v_j < 0$ that can pay only a part of their liabilities to other banks. The ratio is defined as follows:

$$\theta_j := \frac{\sum_{m=1}^{N} \Pi'_{jm} p_m + e_j}{d_j} \in (0, 1). \tag{5}$$

Under this assumption, only the banks identified in Step 2 would default. We replace l_{ij} with $\theta_j l_{ij}$ to ensure the limited liability criterion is met. Thus, we obtain new l_{ij}, Π_{ij}, d_i, and v_i. For example, when l_{ij} is replaced with $l_{ij}^{new} := \theta_j l_{ij}$ $(i = 1, \ldots, N;\ j = \text{a fixed number})$, the new Π is as follows:

$$\Pi_{new} = \begin{bmatrix} \frac{l_{11}}{d_1^{new}} & \cdots & \theta_j \frac{l_{1j}}{d_1^{new}} & \cdots & \frac{l_{1N}}{d_1^{new}} \\ \vdots & \ddots & \vdots & \iddots & \vdots \\ \frac{l_{i1}}{d_i^{new}} & \cdots & \theta_j \frac{l_{ij}}{d_i^{new}} & \cdots & \frac{l_{iN}}{d_i^{new}} \\ \vdots & \ddots & \vdots & \iddots & \vdots \\ \frac{l_{N1}}{d_N^{new}} & \cdots & \theta_j \frac{l_{Nj}}{d_N^{new}} & \cdots & \frac{l_{NN}}{d_N^{new}} \end{bmatrix}. \tag{6}$$

where the new d_i is $d_i^{new} = l_{i1} + \cdots + (\theta_j l_{ij}) + \cdots + l_{iN}$. Step 2 is repeated for all defaulting banks.

This procedure provides a clearing payment vector for the CDS network that satisfies equation (3). Next, we distinguish between a stand-alone default caused by a guarantor's insolvent situation and a contagious default caused by the defaults of other banks. These two types of defaults are described as follows:

Type 1: Stand-alone default:

$$V_i(T) - D_i(T) := \Big(\sum_{j=1}^{N} \Pi'_{ij} d_j - d_i \Big) + e_i \leq 0. \tag{7}$$

where $V_i(T)$ is the market value of the total assets of bank i at time $t = T$ ($T = 1$ month, 2 months,. . .), and $D_i(T)$ is the total face value of the interest-bearing liabilities with regard to bank i at time $t = T$ and is fixed quarterly. The net claims of bank i are composed of CDS net positive exposures ($\sum_{j=1}^{n} \Pi'_{ij} d_j - d_i$) and non-CDS net exposures (e_i). Bank i falls into stand-alone default if it cannot honor its payments, under the assumption that all of the other banks honor their promises.

Type 2: Contagious default:

$$\Big(\sum_{j=1}^{N} \Pi'_{ij} d_j - d_i\Big) + e_i > 0, \tag{8}$$

$$and$$

$$\Big(\sum_{j=1}^{N} \Pi'_{ij} p^*_j - d_i\Big) + e_i \leq 0. \tag{9}$$

A contagious default occurs when bank i's net claims are positive (equation (8)) but other banks cannot fulfill their promises to the bank (equation (9)).

The estimation for the non-CDS net exposures in equation (7) is provided as follows:

$$e_i = V_i(T) - D_i(T) - \Big(\sum_{j=1}^{n} \Pi'_{ij} d_j - d_i\Big). \tag{10}$$

Because both net CDS exposures ($\sum_{j=1}^{n} \Pi'_{ij} d_j - d_i$) and total liabilities $D_i(T)$ are constant, and total asset value $V_i(T)$ is a random variable, net non-CDS exposures (e_i) is a random variable as well. Therefore, we first estimate the market value of $V_i(T)$ ($T = 1$ month, 2 months,...) in a multi-period setting using equity data.

4. Data

The financial data used in our research, including market capitalization data, are obtained from the Bankscope database provided by Bureau van Dijk. In addition, the data for the CDS contracts come from Call Reports,[6]

[6]In the literature review, we mention Peltonen et al. (2014) as an example of using the dataset from the DTCC. The DTCC dataset contains the names of reference entities, but has no detailed information for the time-series and counterparties. Hence, the DTCC dataset is not available for our network analysis.

as collected by the FDIC under Section 1817(a)(1) of the Federal Deposit Insurance Act. All regulated financial institutions in the United States are required to file periodic financial and other information with regulators and other parties. Each national bank, state member bank, and insured non-member bank is required by the Federal Financial Institutions Examination Council (FFIEC) to file a Call Report.

In the Call Reports, we use two items: gross positive fair value (GPFV) and gross negative fair value (GNFV).[7] GPFV is the sum total of fair values of contracts in which a bank is owed money by its counterparties, without taking the effect of netting and collateral into account, and is adjusted for compression, if any. GNFV is the sum total of fair values of an institution's contracts in which the bank currently has a balance outstanding to the counterparty. In addition, GNFV is the same as GPFV in terms of treatment of netting, collateral, and compression.

In terms of netting, we calculate "netted current credit exposure" from GPFV and GNFV after taking legally enforceable bilateral netting agreements into account. As for collateral, there are no data by counterparty. However, the ISDA used the percentage (69%) of trades covered by collateral agreements rather than the percentage of credit exposure covered by collateral to calculate the impact of collateral on credit exposure (ISDA, 2013). Based on this figure, a significant risk mitigation effect for collateralization is expected.

Furthermore, in the light of central clearing and compression, the CDS data from the FDIC Call Reports cover bilateral OTC contracts, including ones that are centrally cleared. According to the ISDA OTC Derivatives Market Analysis Year-End 2012, approximately 18% of CDS notional outstanding was eliminated via portfolio compression in 2012 (ISDA, 2013).

The share for the top 22 US banks ranked in terms of fair value amounts is more than 99% of the amounts listed in Call Reports. Nonetheless, AIG is not included in this report because it is not a bank and sold mainly bespoke CDSs.

[7] "Fair value" is a specific type of "market value." It is defined by a legal or regulatory jurisdiction and varies with individual jurisdictions. Therefore, GMV in the BIS statistics of Figure 2 is in accordance with "gross fair value" in the FDIC Call Reports.

5. Estimations

5.1. Bilateral exposures matrix

If all players in the US CDS network are included in the estimation of the bilateral exposures matrix, the bilateral positive fair value matrix would be in perfect accordance with the bilateral negative fair value matrix. According to our data resource—Call Reports by the FDIC—the average difference between both matrices from March 2006 to September 2015 is only 3%. Hence, in the US CDS market assumed in our research, almost all market dealers and other banks are covered. Thus, we estimate the bilateral exposures matrix using two aggregated fair value data and calculate the netted current credit exposure matrix by netting of the bilateral positive fair value matrix and the bilateral negative fair value matrix. Refer to Appendix B for details.

Our dataset is based on fair value data reported by US banks. However, the data with dealer banks are not subdivided into the data with CCP-related trades, such as dealer–CCP trades, and data with non-CCP-related trades, such as dealer–dealer trades and dealer–non-dealer bank trades, in FDIC Call Reports. Because the counterparty risk to CCPs is currently almost zero,[8] the gross fair values with CCP-related trades need to be excluded from the gross fair values with dealer banks. The average market share with CCP-related trades in terms of GMV is indicated by the dotted line labeled as CCPs—GMV in Figure 2. Thus, for example, if the gross fair value reported by a dealer bank in a quarter is 100 billion US dollars and the share with CCPs in the year is 10%, the weight-averaged gross fair value is then calculated as 90 billion US dollars ($= 90\text{billion}\times 1 + 10\text{billion}\times 0$).

5.2. Market value of assets

We need to consider the market value of assets, volatilities, and drifts in order to calculate the probabilities of default in the context of the structural model approach of credit risk. Because the asset value is a latent variable, it cannot be calculated directly. Hence, we estimate the market value-based parameters of assets using the estimation procedure applied by Duan (1994, 2000), Crosbie and Bohn (2003), and Duan et al. (2004). Refer to Appendix C for the details.

[8]CCPs have failed in the past and hence, have been shown to be potentially dangerous (Gregory, 2014).

Table 2: Descriptive statistics of bilateral exposures (in 1,000 US dollars)

percentile	Dec-07	Dec-08	Dec-09	Dec-10	Dec-11	Dec-12	Dec-13	Dec-14
25th	0	0	0	0	0	0	0	0
Median	0	0	0	0	0	0	0	0
75th	1	1	0	0	0	0	0	0
95th	5,821	16,491	3,692	1,586	2,486	616	478	303
Maximum	2,724,729	9,980,327	3,487,037	2,258,791	1,698,595	1,070,212	2,286,276	4,177,477
Mean	16,992	44,626	23,837	11,984	10,871	8,292	10,459	13,760
S.D.	164,975	518,288	218,286	121,428	99,741	71,607	118,068	200,782

Note: The sum of the bilateral exposures each year is 484 ($= 22^2$). S.D. stands for standard deviation.

6. Results

6.1. Estimation of bilateral CDS exposures matrix and network analysis

We estimate the bilateral netted CDS exposures matrix Z expressed in equation (B.5) before we conduct the various systemic risk analyses. A bank's "gross negative fair value" is allocated as the CDS exposures among the counterparties. The descriptive statistics of the estimated matrices are shown in Table 2, and the percentile distribution by quarter is shown in Figure 3. Table 2 shows the quartile at any year-end. All of the exposure sizes are near zero at the 75th percentile. However, the sizes increase sharply from the 95th percentile to the maximum, and they range from 1 billion to 10 billion US dollars at maximum.

Next, we focus on the analyses of the US OTC CDS bilateral market without central clearing, as shown in left panel of Figure 1 using network centrality measures. The trade shares indexed as dealers/other banks in the market are shown in Figure 2. The proportion of the bilateral market has been decreasing gradually.

In order to consider the applicability of the various centrality measures to the interconnectedness in the US CDS network, we calculate the correlation among eight selected centrality measures: degree, weighted degree, eccentricity, closeness centrality, eigenvector centrality, betweenness centrality, hyperlink-induced topic search (HITS) hub centrality, and PageRank (Figure 4). The centrality measures shown for illustrative purposes are calculated based on the data as at the end of 2007 to 2014 and September 2015, and some are substantially different from one another. The Pearson correlation is reported in the upper part, and each line shows the linear regression.

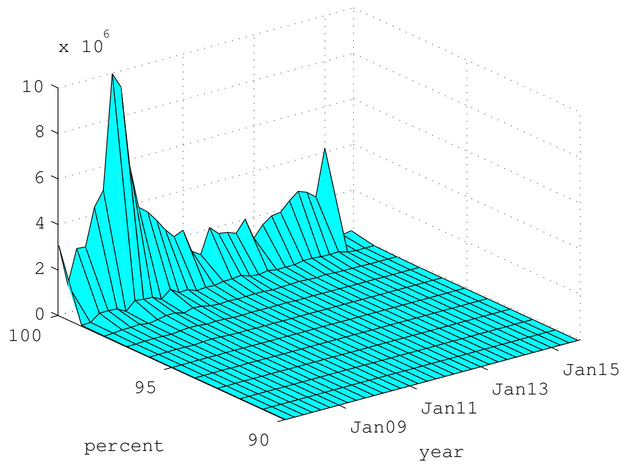

Figure 3: Bilateral exposure size distribution

Note: The distribution is shown in the range from the 90th percentile to the 100th percentile.

Eccentricity and closeness centrality represent a bank's closeness, whereas betweenness centrality measures a bank's substitutability and shows the bank's central role in the network. The HITS hub centrality, eigenvector centrality, and PageRank (a variation of eigenvector centrality) capture the magnitude of the network relationships. The first measure relates to the obligation payments and shows the systemic importance in the network, whereas the second and third measures relate to the claims and exhibit no systemic importance. From Figure 4, some strong correlation relationships are observable, such as eccentricity versus closeness centrality and betweenness centrality versus weighted degree. In the following part of this subsection, in light of each centrality's availability in terms of systemic importance, we analyze the interconnectedness, focusing on degree, betweenness centrality, and HITS hub centrality in more detail.

First, the "degree" of a node is the number of edges connected to the node. The "out-degree" is the number of outgoing edges emanating from a node, and the "in-degree" is the number of incoming edges onto a node. In a directed graph, which is defined as a set of nodes in which all the edges are directed from one node to another, each node has a maximum of two

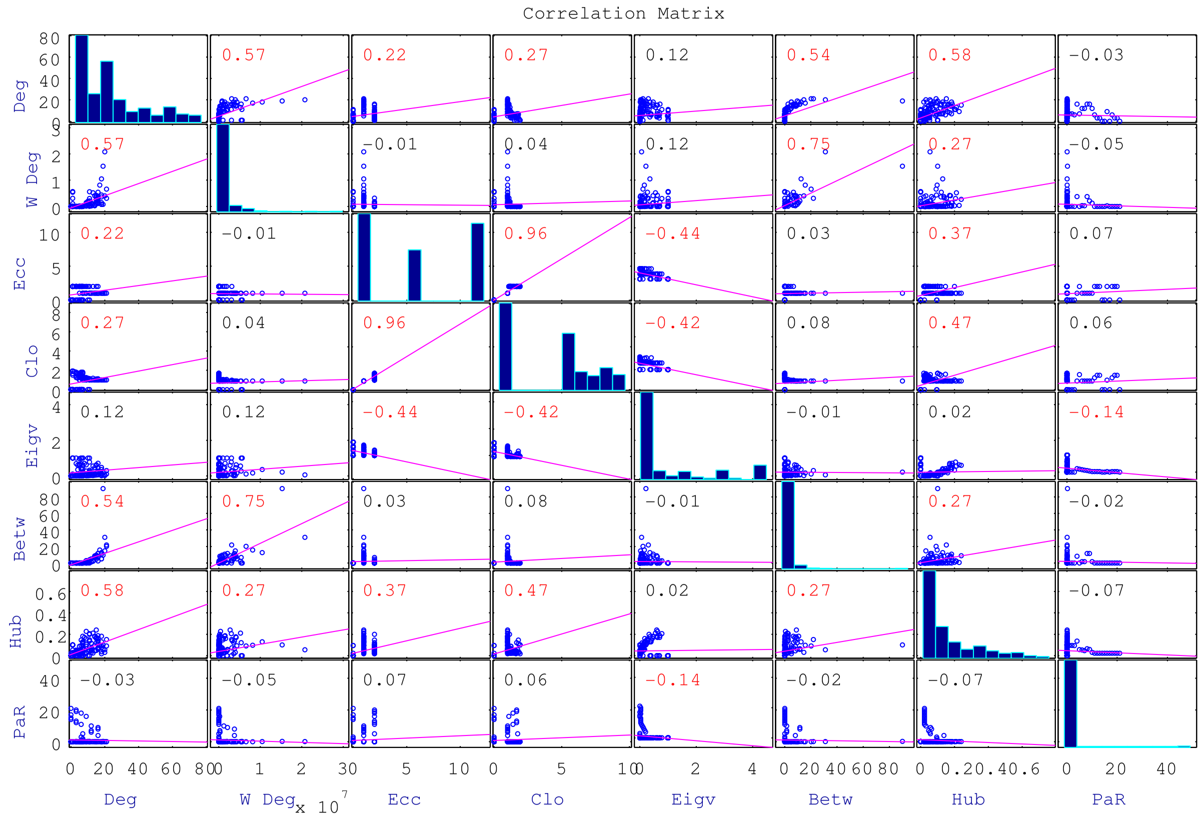

Figure 4: Correlation plot of network centrality measures

degrees for each edge. The total degree of a node is the sum of its in- and out-degrees. However, it is to be noted that the degree of each node in a graph after taking bilateral netting is one at a maximum. The degree of a node can be considered as a proxy variable for interconnectedness.

The top five banks ranked according to interconnectedness, which is measured in terms of the degree of their nodes, are shown in the upper part of Table 3. They are dealers, such as JPMorgan Chase, Bank of America, Citibank, HSBC Bank USA, and Goldman Sachs. Their netted degrees (i.e., one way) reached a peak of 15–21 for the crisis period of 2007 to 2009 and thereafter, decreased. This trend is confirmed in Figure 5, which denotes the directed graphs of the US CDS network as at the end of 2007 to 2014 and September 2015.[9] The width of an edge denotes the size of its netted CDS exposures at the end of the quarter. The color is a mix of its source (start) node color and its target (end) node color. Certainly, the network is significantly dense for the period, and thereafter, is sparser. In addition, we can confirm that three to six large banks play a central role in each directed graph.

Second, we analyze the interconnectedness of the US CDS network in terms of "betweenness centrality" (Krause and Giansante, 2012; Kanno, 2015), which is a centrality measure of a node within a network graph. This measure quantifies the number of times a node acts as a bridge along the shortest path between two other nodes. Hence, betweenness centrality can also be considered as a measure of substitutability. A node with high betweenness centrality could potentially influence the spread of information through the network. If the normalized betweenness centrality, which is defined as $(bc - \min(bc))/(\max(bc) - \min(bc))$ (bc: the betweenness centrality of a node), is close to one, a node (i.e., bank A) acts as a bridge along most of the shortest paths connecting two other nodes (i.e., banks B and C). If it is close to zero, bank A is less important to the two other banks (i.e., banks B and C) (Kanno, 2015).

The top five banks ranked according to interconnectedness as measured in terms of betweenness centrality are shown in the middle part of Table 3. There are about five banks a year with betweenness centrality of more than zero, and the original (not normalized) centrality measure level is around 1.6

[9]For convenience drawing the graph, the exposure data are truncated to 10,000 US dollars.

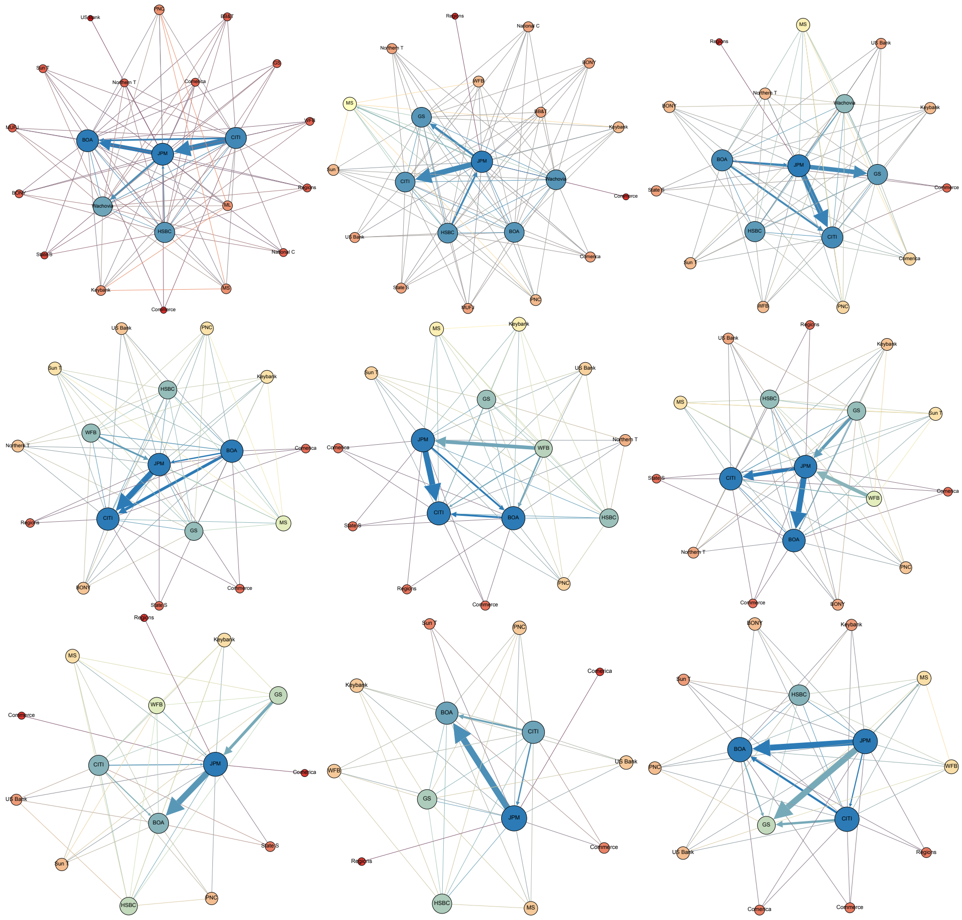

Figure 5: Directed graphs based on degrees

Note 1: The figures are for 2007 to 2009 from the upper-left panel to the upper-right panel, for 2010 to 2012 from the middle-left panel to the middle-right panel, and for 2013 to 2015 from the lower-left panel to the lower-right panel.

Note 2: The graph is drawn in the Fruchterman–Reingold layout. The nodes are the mass particles, and the edges are the springs between the particles.

on average and 90 at maximum. This is because of the structure that three to six banks play a dominant role in the US CDS network. If a bank with high betweenness centrality defaults, the clearing payment in the US CDS market may become non-functional.

Third, we analyze the interconnectedness of the US CDS network in terms of "HITS hub centrality." HITS is known as "hubs" and "authorities." HITS is proposed to find the main structures in the World Wide Web (WWW). Web pages are divided in two categories: hubs and authorities. By the creation of a hyperlink from page p to q, the author of page p increases the authority of q. The authority of a WWW site would be to consider its "in-degree" (i.e., the hyperlinks to return to the home page). Hence, HITS authority centrality is not suitable for measuring systemic importance of banks in the CDS network. By contrast, a hub is defined as a WWW site pointing to many authorities. Hence, HITS hub centrality considers systemic importance of banks in terms of hub scores based on its "out-degree." Banks with the highest hub play a central role in the network. The weights are normalized to ensure that the sum of their squares is 1.

The top five banks ranked according to interconnectedness as measured in terms of HITS hub centrality are shown in the lower part of Table 3. For the period of 2007 to 2010, in addition to dealer banks, such as JPMorgan Chase, Bank of America, Citibank, and Goldman Sachs, non-dealer banks, such as PNC and Keybank, are selected in terms of HITS hub centrality. Thereafter, dealer banks are mainly selected as hub banks. Furthermore, the correlation between HITS hub centrality and degree centrality is 0.58 from Figure 4 and thus, is moderately high. As a result, the top five banks list of the HITS hub centrality is relatively similar to that of degree centrality.

6.2. Estimation of market variables

The market values of assets and the drifts and volatilities of the asset returns[10] are estimated for each bank using the maximum likelihood estimation methodology detailed in Appendix C. All of the parameter estimations are conducted in local currency units. After final consolidation of the data, the values in local currency units are converted into US dollars using quarterly foreign exchange rates.

[10]Although the estimation frequency of assets and volatilities is monthly, one of the drifts is estimated yearly, given the estimation procedure. In the later default analysis in Subsection 6.3, only asset values are used.

Table 3: Top 5 banks ranked according to interconnectedness measured in terms of degree, betweenness centrality, and HITS hub centrality

Centrality	R	Dec-07	Dec-08	Dec-09	Dec-10	Dec-11	Dec-12	Dec-13	Dec-14	Sep-15
Degree	1	JPM (21)	JPM (20)	JPM (17)	JPM (16)	JPM (15)	JPM (16)	JPM (14)	JPM (13)	JPM (14)
	2	BOA (21)	BOA (18)	BOA (16)	BOA (16)	BOA (15)	BOA (16)	BOA (11)	BOA (11)	BOA (14)
	3	CITI (20)	CITI (18)	CITI (16)	CITI (16)	CITI (15)	CITI (16)	CITI (11)	CITI (11)	CITI (14)
	4	HSBC (19)	HSBC (18)	HSBC (15)	HSBC (12)	HSBC (11)	HSBC (12)	HSBC (9)	HSBC (9)	HSBC (11)
	5	WB (18)	WB (18)	GS (15)	GS (12)	GS (11)	GS (12)	GS (9)	GS (9)	GS (9)
			GS (18)		WFB (12)					
BC	1	BOA (22)	JPM (31)	JPM (16)	JPM (12)	JPM (9)	GS (5)	JPM (9)	CITI (4)	CITI (8)
	2	JPM (20)	CITI (13)	GS (8)	BOA (12)	BOA (9)	JPM (3)	GS (3)	JPM (2)	JPM (8)
	3	CITI (15)	BOA (5)	CITI (7)	CITI (8)	CITI (9)	BOA (3)	BOA (3)		BOA (7)
	4	WB (11)	WB (5)	BOA (7)	GS (2)	GS (1)	CITI (3)	CITI (3)		HSBC (3)
	5	HSBC (10)	HSBC (5)	HSBC (5)	HSBC (2)	HSBC (1)	HSBC (1)	WFB (1)		GS (1)
			GS (5)		WFB (2)		WFB (1)			
HITS	1	PNC (0.18)	PNC (0.13)	GS (0.17)	CITI (0.14)	CITI (0.24)	BOA (0.19)	BOA (0.21)	JPM (0.21)	GS (0.23)
	2	WB (0.18)	Keybank (0.13)	MS (0.17)	Keybank (0.1)	GS (0.2)	CITI (0.16)	JPM (0.18)	GS (0.16)	BOA (0.18)
	3	BOA (0.15)	CITI (0.13)	Keybank (0.15)	PNC (0.1)	BOA (0.16)	JPM (0.14)	CITI (0.15)	HSBC (0.11)	HSBC (0.14)
	4	Keybank (0.12)	WFB (0.13)	CITI (0.12)	US Bank (0.09)	JPM (0.12)	HSBC (0.12)	HSBC (0.09)		CITI (0.09)
	5	JPM (0.09)	Sun T (0.11)	JPM (0.1)	JPM (0.06)	HSBC (0.08)	GS (0.07)	Sun T (0.09)		

Note 1: The figures in parentheses indicate degree centrality, betweenness centrality, or HITS hub centrality.

Note 2: Abbreviations: R: Ranking; BC: Betweenness centrality; HITS: HITS hub centrality; JPM: JPMorgan Chase; BOA: Bank of America; CITI: Citibank; GS: Goldman Sachs; Sun T: Sun Trust; WFB: Wells Fargo; WB: Wachovia.

The six large banks (dealers) designated as global systemically important banks (G-SIBs) are selected as graph samples from 22 banks. Their estimation results are shown in Figures 7, 8, and 9.

Because the asset value is a latent variable[11] and is not observable in the financial market, it is important to check its level and time variation. Figure 7 shows that the asset values of previous investment banks, such as Goldman Sachs and Morgan Stanley, substantially decreased just after the bankruptcy of Lehman Brothers. By contrast, four large commercial banks increased their asset values.

As for the asset return volatilities, in general, the higher the asset volatility of a bank, the larger is its probability of default in our framework—the structural model approach of credit risk. The quarterly values of six selected banks are less than 10%, which is not very volatile. Finally, with respect to the asset return drifts, the quarterly average of 22 banks for 2006 to 2015 is about 0.03%. The less the drift of a bank, the larger is its probability of default in our framework.

6.3. Contagious default analysis

The theoretical number of stand-alone defaulting banks and contagious defaulting banks is estimated. During the estimation period of 2006–2015, many stand-alone defaults occurred. By contrast, one contagious default occurred. Figure 10 indicates the monthly variations of the total number of stand-alone defaulting banks (upper panel) and contagious defaulting banks (lower panel). Shortly after the bankruptcy of Lehman Brothers, the number of stand-alone defaults jumped significantly, peaking at 10 in March 2009. In addition, a second peak can be seen around March 2008 (the period of the subprime mortgage crisis).

Figure 11 indicates the monthly variations of the loss amounts of banks listed in Table A.6. The banks that suffered losses included the defaulting banks as well as some banks that did not fall into a stand-alone default or a contagious default category. Each bank suffered loss for any period from 2006 to 2015, and during the subprime mortgage crisis and shortly after the bankruptcy of Lehman Brothers, five large dealer banks—JPMorgan Chase, Bank of America, Citibank, Goldman Sachs, and HSBC Bank USA—suffered losses as a result of the payments from defaulting counterparties.

[11]The banks' market capitalization data are not published in Bankscope for the period when asset values were zero.

This result is in accordance with the reality. These five banks had credit exposure related to their derivatives trading that exceeds their capital, with four in particular—JPMorgan Chase, Goldman Sachs, HSBC Bank USA, and Citibank—taking especially large risks. According to the Office of the Comptroller of the Currency, at the end of 2008, Bank of America's total credit exposure to derivatives was 179% of its risk-based capital; Citibank's was 278%; JPMorgan Chase's was 382%; and HSBC Bank USA's was 550%. In addition, in the fourth quarter of 2008, Goldman Sachs began reporting as a commercial bank, revealing an alarming total credit exposure of 1,056%, or more than 10 times its capital.

6.4. Stress test

We conduct a systemic stress test to verify the resilience of the US CDS network at an evaluation point in the future. CDS market dealers are assumed as systemic sources with the potential to trigger systemic contagious risk in the test. In addition, it is meaningful to examine whether the increasing share of contracts through CCPs reduces systemic contagious risk. Hence, we examine whether any one or more of the listed banks trigger contagious defaults.

To examine the effect of contagious defaults, we stress any one or more banks in each test run. The evaluation time point is assumed as the end of 2016, at which point the market value of the asset of the selected bank is reduced by 30% from its value at the end of September 2015. In terms of the network structure, the exposures matrix at the end of either March 2009 (during the global financial crisis, without CCPs) or September 2015 is assumed. The liability value of each bank is assumed the same as that at September 2015.[12] Hence, the stresses are imposed on the asset value of each bank and/or the CDS exposures network.

As a result, in a gross payment case (i.e., without bilateral netting), one contagious default is triggered by stand-alone defaults of four major dealers, given the gross exposures matrix at the end of March 2009, whereas no default is triggered for the exposures matrix under consideration of netting at the end of March 2009 and at the end of September 2015. In addition, no default is triggered by the stand-alone default of one major dealer for each

[12]Because a bank's GNFV for the CDS outstanding to its asset value is so small, being 2.5% at maximum, its recovery payment defaults to its counterparties would not trigger a contagious default.

Table 4: Number of contagious defaulting banks based on the stress test

Trigger dealer(s) (no. of dealers)	CDS exposures matrix	
	March 2009	Sep 2015
BOA & CITI & GS & HSBC (4)	1(Gross)/0(Netting)	0
BOA / CITI / GS / HSBC / JPM (1)	0	0

Note 1: Abbreviations: BOA: Bank of America; CITI: Citibank; GS: Goldman Sachs; HSBC: HSBC Bank USA; JPM: JPMorgan Chase.

Note 2: "Gross" in parentheses indicates the use of gross exposures matrix whereas "Netting" indicates the use of netted exposures matrix.

exposures matrix (Table 4). Judging from this result, unless many stand-alone defaults transpire simultaneously in a severe economic environment, such as the global financial crisis, contagious default is unlikely. It is even more unlikely because the share for central clearing steadily increases in the near future. In addition, CDS payment defaults would not trigger contagious defaults directly, given the current CDS network structure.

7. Sensitivity analysis

We assess the robustness of our analyses based on the estimated bilateral exposures matrix. In Subsection 5.1, we estimate the bilateral exposures matrix without any restriction as to the distribution for exposures, although we consider the risk mitigation effect by CCPs after March 2010. However, in this section, we consider the information pertaining to the core–periphery structure as additional information for more possible estimation.[13]

[13] As a relevant study, Mistrulli (2011) argues that the comparison between the bilateral exposures matrix based on the estimation methodology detailed in Appendix B (i.e., maximum entropy method) and the observed interbank matrix can be interpreted as a theoretical comparison between complete and incomplete markets, which is proposed by Allen and Gale (2000). In addition, Paltalidis et al. (2015) compare results obtained from the maximum entropy method and the actual bilateral exposures matrix for the German and French banking networks. They conclude that the maximum entropy method neither over- nor under-estimates the bilateral exposures and is a suitable way to calibrate losses generated by systemic shocks.

Craig and von Peter (2014) present a core–periphery network model. They call the set of top-tier banks "the core" and the set of lower-tier banks "the periphery." In the interbank market, top-tier banks lend to each other (CC[14]: core to core), top-tier banks borrow from lower-tier banks (PC: periphery to core), top-tier banks lend to lower-tier banks (CP: core to periphery), and lower-tier banks do not lend to each other (PP: periphery to periphery; a square matrix of zeros.). Thus, the bilateral exposures matrix Y with the core–periphery structure is constructed as follows:

$$Y = \begin{bmatrix} CC & CP \\ PC & PP \end{bmatrix}. \tag{11}$$

Core banks in the interbank market can be regarded as money center banks. They act as dealers in a broad range of markets, including the CDS market. According to Langfield et al. (2014), the strength of the core–periphery structure significantly varies depending on the asset class; hence, the core–periphery model fits more strongly for derivatives and marketable securities than for unsecured lending and repo agreements.

In addition, Langfield et al. (2014) argue that pure derivatives houses are at the core of the cluster related to "net CDS sold" as well as lending, marketable securities, securities lending and repo exposure, and derivatives exposure. This cluster comprises a group of dealers with significant exposures to securities holders, diversified banks, and other banks in their own cluster, mostly in the form of derivatives. These derivative houses are most likely exposed to securities holders, whereas they have exposures in both directions with more diversified banks partly due to market-making activities.

In order to obtain the optimal matrix with the core–periphery structure, the approach to minimize a distance measure of the total error score is proposed in Craig and von Peter (2014). As a result of the calculation by our optimization, five or six major banks[15] are selected as core banks. Figure 6 denotes the core–periphery network structure as at the end of 2007 and 2008.

Compared with Figure 5, there is no non-dealer–non-dealer relationship,

[14] Its symbol indicates a block in the following matrix Y.

[15] As at the end of 2007 (prior to the bankruptcy of Lehman Brothers), JPMorgan Chase, Citibank, Bank of America, HSBC, and Wachovia are selected. As at the end of 2008 (shortly after the bankruptcy of Lehman Brothers), Goldman Sachs is selected in addition to the five banks.

although the related trade volume is essentially very small. We render the network with the core–periphery structure sparser than the true network because it is assumed no contracts exist between periphery banks. By contrast, the estimated network is denser than the true network because of the estimation algorithm detailed in Appendix B. We use a modified version of the Jaccard index and network density as means for validating the structure in the network.

First, we introduce the so-called Jaccard index as a means to measure similarities between two network structures. The index counts the number of linkages that appear in two CDS networks (i.e., the network by an unconstrained estimation and the network with the core–periphery structure) and relates it to the total number of linkages in both networks. We make one minor modification to the index to compensate for the deficiency—the modified Jaccard index ranges from 0 to 1. Refer to Appendix D for details. Table 5 denotes the modified Jaccard index both prior to the bankruptcy of Lehman Brothers (as at December 2007) and shortly after the bankruptcy of Lehman Brothers (as at December 2008). Both networks are quite similar in that the index figures are 100% for both periods. Hence, the network obtained from the unconstrained estimation has the same features and linkages as the network with the core–periphery structure.

Second, we calculate the network density, which is the ratio of actual to potential links between the nodes (Clerc et al., 2014). In a directed network, the ratio is defined as:

$$p = \frac{m}{n(n-1)} \tag{12}$$

where n is the number of nodes and m is the number of linkages connecting the nodes. This ratio ranges from 0 to 1, with higher values denoting "denser" networks. As shown in Table 5, the network density for the network obtained from the unconstrained estimation in Subsection 6.1 is only a little larger than that for the network with the core–periphery structure as at both dates, and hence, the number of degrees of this network is equal to or less than that of the true network.

Thus, in terms of similarities between the network obtained from the unconstrained estimation and the true network, we are convinced that our analyses are robust as a whole.

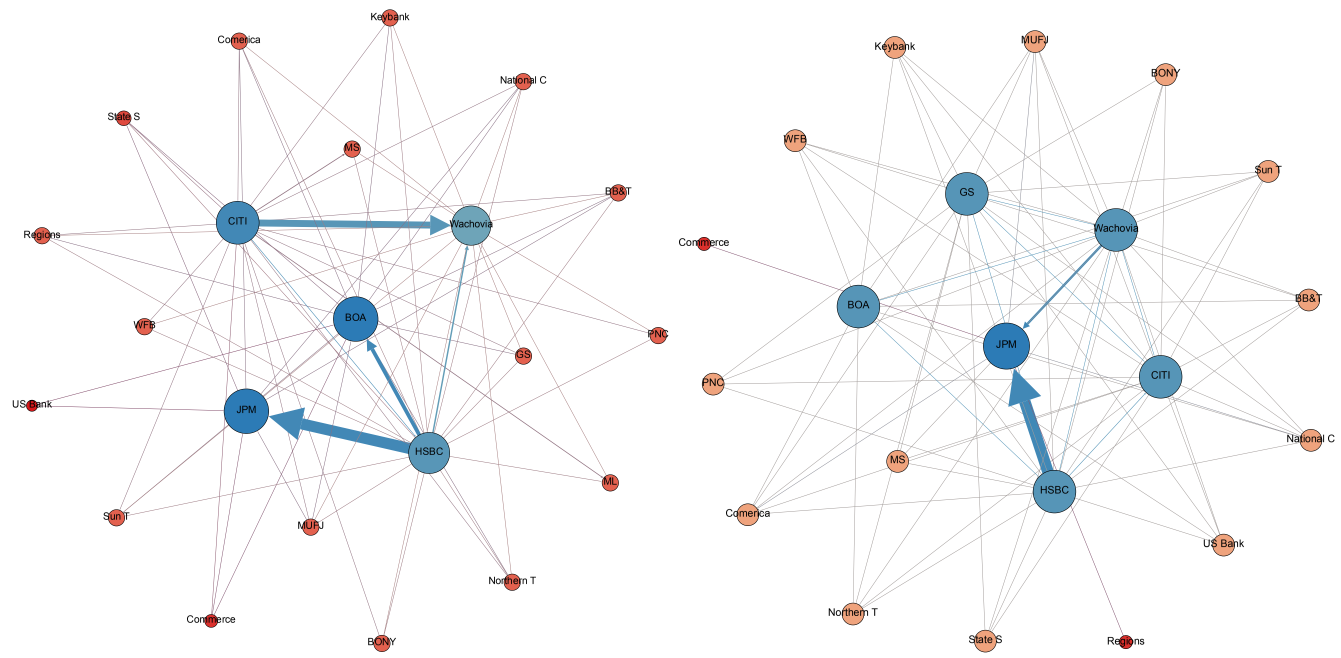

Figure 6: Directed graphs with the core–periphery network structure as at the end of 2007–2008

Note: The left and right panels are as at the end of 2007 and 2008, respectively.

Table 5: Modified Jaccard index and network density

Date	Mod. JC Index	Network density		
		Unc.	CP	Diff. (Unc.−CP)
Prior to bk of LB (Dec-07)	100.00%	20.1%	19.3%	0.8%
Shortly after bk of LB (Dec-08)	100.00%	23.6%	22.6%	1.0%

Note : Abbreviations: JC: Jaccard; Unc.: the density for the network obtained from the unconstrained estimation; CP: the network density for the network with the core–periphery structure; bk of LB: the bankruptcy of Lehman Brothers.

8. Conclusions

In this study, we analyzed the network structure of the US CDS market considering bilateral netting, central clearing, and compression and assessed the systemic importance of each bank for the period of the global financial crisis and thereafter. During the crisis, AIG faced a management crisis owing to AIG-FP's massive short position. Nonetheless, the interconnectedness in the US CDS network is not necessarily large compared to that of other financial networks owing to the introduction of CCPs.

First, we theoretically analyzed the CDS network structure using various network centrality measures, in terms of assessing systemic importance. Based on such measures as degree, betweenness centrality, and HITS hub centrality, one significant finding is that three to six major banks have played a central role in the network in the past.

Second, we modeled the mechanism of contagious defaults in the US CDS network and theoretically analyzed the contagious defaults conditional on a stand-alone default during and after the global financial crisis, using real contracts data on the FDIC Call Reports. Our analysis theoretically shows a few contagious defaults triggered by stand-alone defaults during the global financial crisis.

Third, we conducted a stress test and analyzed the possibility of contagious defaults conditional on any one or more stand-alone defaults in the future. We proved that the possibility of contagious defaults triggered by the risk contagion via the CDS network is very low.

As a complement to the first contribution, sensitivity analysis proved the robustness of the estimated bilateral exposures matrix after netting. Because it is virtually impossible to obtain a complete dataset of the bilateral

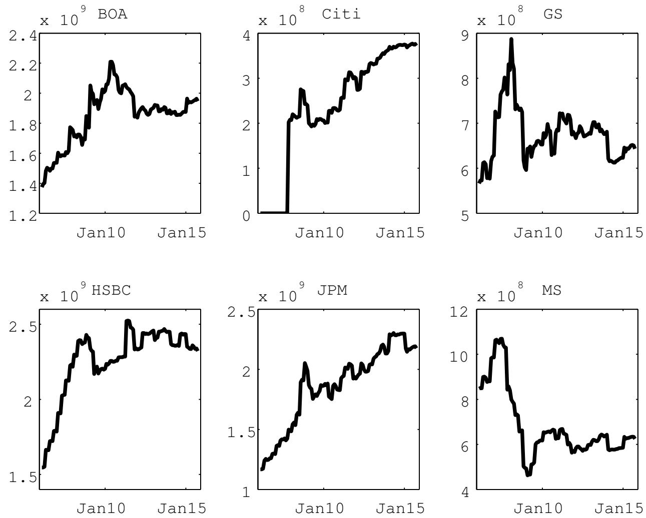

Figure 7: Monthly variations in asset values for selected banks (in 1,000 of local currency units)

exposures for derivative contracts, such as CDSs, we estimated the bilateral exposures matrix from the aggregated positions of each bank using an estimation algorithm (i.e., the RAS algorithm). To assure the robustness of the estimated matrix, we used the core–periphery model and such measures as the modified Jaccard index and network density.

Our methodology could assist in the development of monitoring and early-warning indicators related to systemic risk in the CDS network by supervisory authorities. In addition, it could be used in the implementation of banks' internal systemic stress tests of contagion risk.

Acknowledgments

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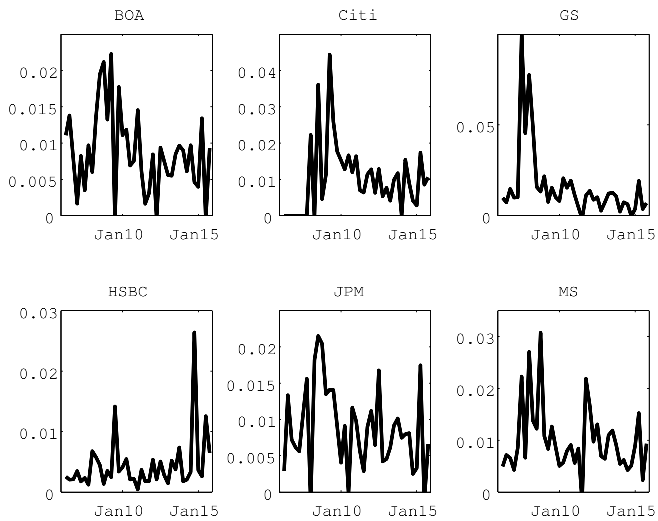

Figure 8: Quarterly variations in asset return volatilities for selected banks

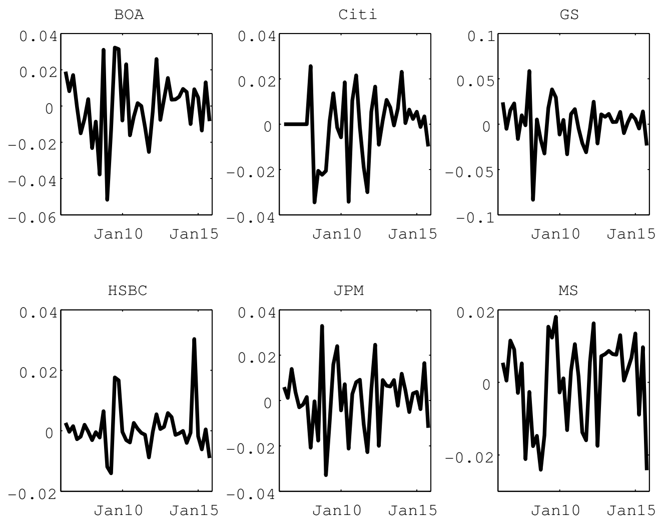

Figure 9: Quarterly variations in asset return drifts for selected banks

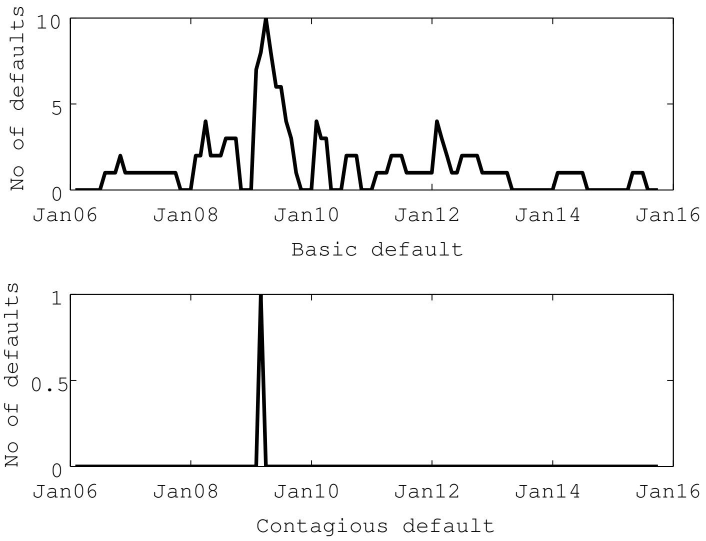

Figure 10: Time variations in number of defaulting institutions

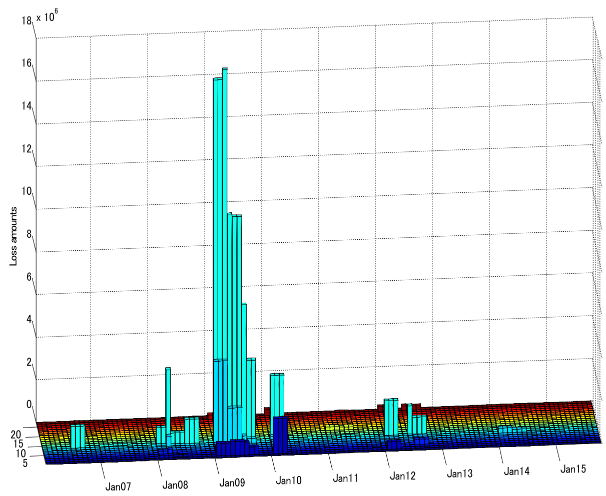

Figure 11: Time variations in loss amounts (in 1,000 US dollars)

Note: The legend number in the vertical axis corresponds to the number in Table A.6.

Appendix A. US banks list

The US banks for the analyses are listed in Tables A.6.

Table A.6: 22 US banks list

No	Bank name	G14	ICE	CME
1	BB & T			
2	Bank of America	○	○	○
3	BONY			
4	Citibank	○	○	○
5	Comerica			
6	Commerce Kansas City			
7	Goldman Sachs	○	○	○
8	HSBC Bank USA	○	○	○
9	JPMorgan Chase	○	○	○
10	Keybank			
11	Merrill Lynch	○	○	○
12	Bank of Tokyo-Mitsubishi UFJ Trust			
13	Morgan Stanley	○	○	○
14	National City			
15	Northern Trust			
16	PNC			
17	Regions			
18	State Street			
19	Sun Trust			
20	U.S. Bank			
21	Wachovia	○		
22	Wells Fargo	○	○	

Note: "○" indicates a dealer in G14, ICE Clear Credit or CME Group. "G14" indicates the largest 14 derivatives dealers.

Source: Chen et al. (2011), ICE Clear Credit (2015), and CME Group Home Page.

Appendix B. Estimation methodology of bilateral exposures matrix

The contract relationship in the US CDS network is represented by the following ($N \times N$) gross negative fair value matrix[16] X:

$$\begin{array}{rcc} & & \sum_j \\ X = & \begin{bmatrix} x_{11} & \cdots & x_{1j} & \cdots & x_{1N} \\ \vdots & \ddots & \vdots & \iddots & \vdots \\ x_{i1} & \cdots & x_{ij} & \cdots & x_{iN} \\ \vdots & \ddots & \vdots & \iddots & \vdots \\ x_{N1} & \cdots & x_{Nj} & \cdots & x_{NN} \end{bmatrix} & \begin{matrix} a_1 \\ \vdots \\ a_i \\ \vdots \\ a_N \end{matrix} \\ \sum_i & \begin{matrix} l_1 & \cdots & l_j & \cdots & l_N \end{matrix} & \end{array} \tag{B.1}$$

where x_{ij} denotes the outstanding exposures payable of bank i to bank j. Summing across row i gives bank i's total gross positive fair value receivable, and summing down column j gives bank j's total gross negative fair value payable as follows:

$$a_i = \sum_j x_{ij}, \; l_j = \sum_i x_{ij}. \tag{B.2}$$

Typically, a bank's aggregated data on gross fair values are obtained only from the FFIEC Central Data Repository; hence, estimating matrix X without imposing further restrictions is not possible. If additional information is unavailable, one possible approach would be to choose a distribution that minimizes the uncertainty, such as the amount of information related to the distribution for these exposures. By following a normalization such that $\sum_i a_i = \sum_j l_j = 1$, the solution $x_{ij} = a_i * l_j$ is yielded, which represents the normalized amount bank i received from bank j. Thus, the exposures reflect the relative importance of each bank in the CDS network. When calculating matrix X, we consider the fact that an bank cannot have exposure to itself.

[16]Matrix X is treated as not only the gross negative fair value matrix L in Section 3 but also as the gross positive fair value matrix.

Therefore, we populate the values into x_{ij}^0 as follows:

$$x_{ij}^0 = \begin{cases} 0 & \text{, for any } i = j \\ a_i l_j & \text{, otherwise.} \end{cases} \tag{B.3}$$

Matrix $X^0 = (x_{ij}^0)$ violates the summing constraints expressed in equation (B.2). Hence, a new matrix X must be found to satisfy the constraints. Some possible methodologies are presented by Upper (2011), Elsinger et al. (2002), and Wells (2004). The solution is provided by solving the optimization problem as follows:

$$\begin{aligned} &\min \sum_{i=1}^{N} \sum_{j=1}^{N} x_{ij} \ln \left(\frac{x_{ij}}{x_{ij}^0} \right) \\ &\text{subject to } \sum_{j=1}^{N} x_{ij} = a_i, \ \sum_{i=1}^{N} x_{ij} = l_j, \ x_{ij} \geq 0. \end{aligned} \tag{B.4}$$

The RAS algorithm is used to solve this type of problem. For further details, refer to Censor and Zenios (1998). Finally, netted current credit exposure matrix Z is calculated as a square matrix whose diagonal elements are all equal to zero, as follows:

$$Z = \begin{bmatrix} 0 & (x_{12} - x_{21})^+ & \cdots & (x_{1j} - x_{j1})^+ & \cdots & \cdots & (x_{1N} - x_{N1})^+ \\ (x_{21} - x_{1,2})^+ & 0 & \cdots & (x_{2j} - x_{j2})^+ & \cdots & \cdots & (x_{2N} - x_{N2})^+ \\ \vdots & \vdots & \ddots & \vdots & \iddots & \vdots & \vdots \\ (x_{i1} - x_{1i})^+ & \cdots & \cdots & 0 & \cdots & \cdots & (x_{iN} - x_{Ni})^+ \\ \vdots & \vdots & \iddots & \vdots & \ddots & 0 & \vdots \\ (x_{N1} - x_{1N})^+ & (x_{N2} - x_{2N})^+ & \cdots & (x_{Nj} - x_{jN})^+ & \cdots & \cdots & 0 \end{bmatrix} \tag{B.5}$$

where $(a)^+$ denotes the maximum of a and zero.

Appendix C. Estimation methodology of market value based parameters of assets

We consider a probability space $(\Omega, \mathcal{F}, P)$ in which the generated filtration $\mathbb{F} = (\mathcal{F})_{t \in [0,T]}$ satisfies the usual conditions. The dynamics of asset value $V(t)$ follows a geometric Brownian motion:

$$\frac{dV(t)}{dt} = \mu_V dt + \sigma_V dW(t), \ t \in [0, T], \tag{C.1}$$

where μ_V is the constant drift of asset returns, σ_V is the constant volatility of asset returns, and $W(t)$ is the standard Brownian motion. The solution to this equation is obtained as

$$V(T) = V(t)e^{\left(\mu_V - \frac{\sigma_V^2}{2}\right)(T-t)+\sigma_V\sqrt{T-t}z},\ t \in [0,T], \tag{C.2}$$

where z is a standard normal random variable. The market value of equity at time T is given as

$$E(T) = \max[V(T) - D(T), 0], \tag{C.3}$$

where $D(T)$ indicates the default threshold expressed by the constant value of the interest-bearing debt for risk horizon T. Under certain assumptions, the solution to equation (C.3) for equity values in t is given by the Black–Scholes model. We estimate the market value of assets using the methodology proposed by Duan (1994), which was later augmented by Duan (2000) and Duan et al. (2004). This methodology is based on a maximum likelihood estimation. Duan et al. (2004) introduce the log-likelihood equation for the estimation of μ_V, σ_V and $V(th), th = (0, \ldots, kh, \ldots, nh)$ using the observed market values of equity as follows:

$$\begin{aligned} l(\hat{\theta}_V; \hat{V}_i(th)|E(th)) = &-\frac{n}{2}\ln(2\pi\hat{\sigma}_V^2 h) - \frac{1}{2}\sum_{k=1}^{n-1}\frac{\left(\hat{R}(kh) - \left(\hat{\mu}_V - \frac{\hat{\sigma}_V^2}{2}\right)h\right)^2}{\hat{\sigma}_V^2 h} \\ &- \sum_{k=1}^{n}\ln(\hat{V}(kh)) - \sum_{k=1}^{n}\ln(\Phi(d_1)), \end{aligned} \tag{C.4}$$

where $\hat{\theta}_V := (\hat{\mu}_V, \hat{\sigma}_V)$, $h = \frac{1}{12}$year, and Φ is the cumulative distribution function of the standard normal variables. $\hat{V}(u)$ $(u = kh \leq nh = T, 1 \leq k \leq n)$ is estimated as the solution to equation (C.3) using the Black-Scholes model as follows:

$$E(u) = V(u)\Phi(d_1) - D(u)e^{-r(T-u)}\Phi(d_2), \tag{C.5}$$

where

$$d_1 = \frac{\ln\frac{V(u)}{D(u)} + \left(r + \frac{\sigma_V^2}{2}\right)(T-u)}{\sigma_V\sqrt{T-u}}, d_2 = \frac{\ln\frac{V(u)}{D(u)} + \left(r - \frac{\sigma_V^2}{2}\right)(T-u)}{\sigma_V\sqrt{T-u}},$$

and r is the risk-free interest rate. Equity and asset volatilities are related in the following equation provided by Crosbie and Bohn (2003):

$$\sigma_E = \frac{V(u)}{E(u)} \Phi(d_2) \sigma_V, \tag{C.6}$$

where σ_E is the constant volatility of equity returns. The time series of the monthly market value of equity from which the parameter is estimated equals $th = (0, h, 2h, \ldots, nh)$, where $n = 12$ months and $h = 1/12$. Each iteration of the optimization calculation produces a time series of monthly values $\hat{V}(th)$, where the maturity of the liability ranges over $th = (0, h, 2h, \ldots, nh)$. The initial values of $\hat{V}^{(m)}(kh)$ and $\hat{\sigma}_V^{(m)}(kh)$ (m: the m-th iteration of the optimization calculations; $1 \leq k \leq n$) are chosen arbitrarily. However, we set $\hat{V}^{(0)}(kh)$ as $E(kh)$ plus $D(kh)$ using data from the balance sheet of bank i and $\hat{\sigma}_V^{(0)}(kh)$ as $\hat{\sigma}_E E(kh)/\hat{V}^{(0)}(kh)$ from equation (C.6). $E(kh)$ is the market capitalization of bank i at the end of the year. $D(0)$ is the total face value of the interest-bearing debt of bank i at the end of the year. σ_E is estimated from the time-series of the monthly natural logarithms of the returns on bank equity as follows:

$$\hat{\sigma}_E = \sqrt{\frac{1}{n} \sum_{k=1}^{n} (\hat{R}(kh) - \bar{R})^2} \sqrt{12}, \tag{C.7}$$

where

$$\hat{R}(kh) = \ln \frac{\hat{V}(kh)}{\hat{V}((k-1)h)}, \tag{C.8}$$

$$\bar{R} = \frac{1}{n} \sum_{k=1}^{n} \hat{R}(kh). \tag{C.9}$$

The estimation procedure is as follows:

Step 1: Estimate the monthly values of $\hat{V}(kh)$ and $\hat{\sigma}_V(kh)$ ($1 \leq k \leq n$) using the monthly market capitalization data from equations (C.5) and (C.6).

Step 2: Calculate the average values of yearly volatilities using the monthly estimated volatilities $\hat{\sigma}_V(kh)$ and set them as the initial values. Estimate the yearly values of μ_V, σ_V by finding the maximum of equation (C.4).

Step 3: Calculate the monthly values of $\hat{V}(kh)$ once again given $\hat{\sigma}_V$ from equation (C.5).

This procedure allows the estimation of parameters based on the methodology proposed by Duan (1994, 2000) and Duan et al. (2004) using the monthly dataset with a maximum of 12 data points.

Appendix D. Modified Jaccard index

The Jaccard index measures similarities between network structures and neglects the weight of the links. Hence, we modify the index to compensate for this deficiency. The weighted adjacency matrices M^1 and M^2 express the features of two given networks $\mathcal{N}_1$ and $\mathcal{N}_2$, respectively (Hałaj and Kok, 2015). Both networks span on the same set of N nodes as follows:

$$\begin{aligned}
M12 &:= \{(i,j) \in \bar{\mathbb{N}} \times \bar{\mathbb{N}} | (i,j) \in \mathcal{N}_1 \wedge (i,j) \in \mathcal{N}_2\} \\
M10 &:= \{(i,j) \in \bar{\mathbb{N}} \times \bar{\mathbb{N}} | (i,j) \in \mathcal{N}_1 \wedge (i,j) \notin \mathcal{N}_2\} \\
M02 &:= \{(i,j) \in \bar{\mathbb{N}} \times \bar{\mathbb{N}} | (i,j) \notin \mathcal{N}_1 \wedge (i,j) \in \mathcal{N}_2\}
\end{aligned}$$

where $\bar{\mathbb{N}}$ stands for a set $\{1, 2, \ldots, N\}$. The set $M12$ express the number of links overlapped among both network graphs (sets), the sets $M10$ and $M02$ are present in one graph but not in the other. The modified Jaccard index is defined as follows:

$$J(\mathcal{N}_1, \mathcal{N}_2) = \frac{(\#M12) \sum_{(i,j)\in M12} (M^1_{i,j} + M^2_{i,j})}{\sum_{Z\in\{M12,M10,M02\}} \Big((\#Z) \sum_{(i,j)\in Z} (M^1_{i,j} + M^2_{i,j}) \Big)} \tag{D.1}$$

where $\#M12$ and $\#Z$ express the number of entries in the set $M12$ and one in any set of a group ($M12$, $M10$, or $M02$), respectively and $J(\mathcal{N}_1, \mathcal{N}_2) \in [0, 1]$.

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